LECTURE V

THE FORMATION OF GALAXIES

THE STELLAR BIRTHRATE FUNCTION

- RATE OF FORMATION OF STARS OF VARIOUS MASSES

STELLAR BIRTHRATE FUNCTION

$$B(M,t)\,dM\,dt = \psi(t)\,\xi(M)\,dM\,dt$$

$B(M,t)$: NUMBER OF STARS PER VOLUME IN MASS $M, M+dM$ AT TIME $t, t+dt$

$\psi(t)$: STAR FORMATION RATE (SFR)

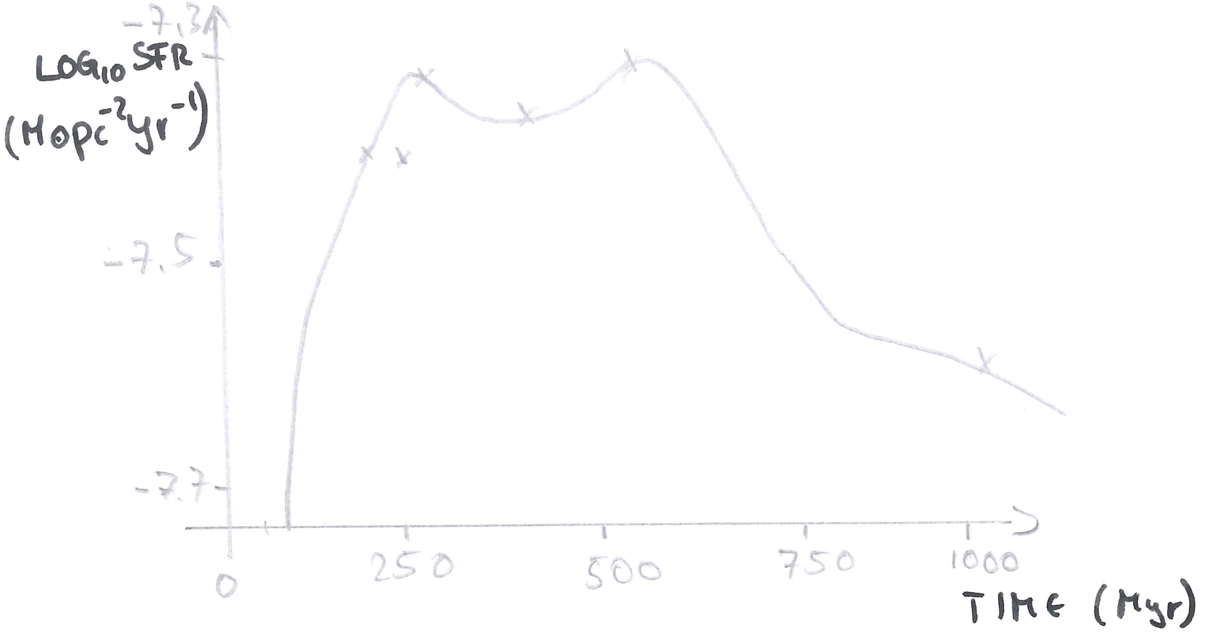
$\xi(M)$: INITIAL MASS FUNCTION (IMF)

SFR: RATE PER UNIT VOLUME WHICH MASS IN THE INTERSTELLAR MEDIUM (ISM) CONVERTED IN STARS

SFR IN GALACTIC DISK: $5 \pm 0.5 \dfrac{M_\odot}{pc^2\,Gyr}$

IMF: RELATIVE NUMBER OF STARS FORMED IN EACH MASS INTERVALL

- FROM SIMULATIONS (BURKERT et al. 1992)



- DECREASES WITH TIME AS AVAILABLE GAS AND DUST IN ISM CONSUMED

MODELLING OF IMF:

$$\xi(M) = \frac{dN}{dM} = C M^{-(1+x)}$$

Salpeter (1955): $x = 1.35$

TODAY: $x = 1.8$ $(7 M_\odot < M_* < 35 M_\odot)$

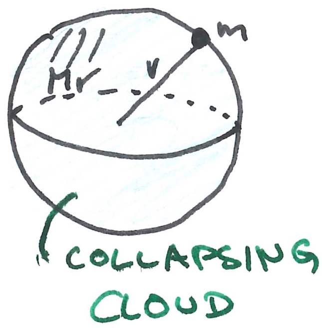
$(\text{FOR } M \gtrsim 40 M_\odot : x \approx 4)$

IMF: NOT YET KNOWN PRECISELY

DISSIPATIVE COLLAPSE MODEL

- FREE-FALL COLLAPSE vs. SLOW, DISSIPATIVE COLLAPSE

↑ FREE-FALL OR DYNAMICAL TIME SCALE



LOCAL ACCELERATION

$$\frac{d^2 r}{dt^2} = - \frac{G M_r}{r^2} \quad (*)$$

- M_r STAYS CONSTANT DURING COLLAPSE

INITIAL DENSITY: ρ_0

" VOLUME : $\frac{4}{3}\pi r_0^3$

$(*) \cdot \frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt}\frac{d^2 r}{dt^2} = - \left(\frac{4}{3}\pi r_0^3 \rho_0\right) \frac{G}{r^2} \frac{dr}{dt}$$

INTEGRATE $\Rightarrow$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \left(\frac{4}{3}\pi r_0^3 \rho_0\right)\frac{G}{r} + C_1$$

C_1: $\frac{dr}{dt} = 0$ INITIALLY AT $r = r_0$

$$\Rightarrow C_1 = - \frac{4}{3}\pi G \rho_0 r_0^2$$

$$\frac{dr}{dt} = - \left[\frac{8\pi}{3} G \rho_0 v_0^2 \left(\frac{r_0}{r} - 1\right)\right]^{1/2}$$

↑ COLLAPSE

SET $\theta = \frac{r}{r_0}$

AND $\varkappa = \left(\frac{8\pi}{3} G \rho_0\right)^{1/2}$

$$\Rightarrow \frac{d\theta}{dt} = -\varkappa \left(\frac{1}{\theta} - 1\right)^{1/2}$$

SET : $\theta \equiv \cos^2 \xi$

$$\Rightarrow \cos^2 \xi \frac{d\xi}{dt} = \frac{\varkappa}{2}$$

INTEGRATE $\Rightarrow$

$$\frac{\xi}{2} + \frac{1}{4} \sin 2\xi = \frac{\varkappa}{2} \cdot t + C_2$$

$r = r_0$ FOR $t = 0 \Rightarrow \theta = 1$

$\Rightarrow \xi = 0$

$\Rightarrow C_2 = 0$

HENCE:

$$\xi + \frac{1}{2} \sin 2\xi = \varkappa \cdot t$$

$t = t_{ff}$ WHEN $r = 0 \Rightarrow \theta = 0$

$\Rightarrow \xi = \frac{\pi}{2}$

$$\Rightarrow t_{ff} = \frac{\pi}{2\varkappa}$$

$$t_{ff} = \left(\frac{3\pi}{32} \frac{1}{G \rho_0} \right)^{1/2}$$

FREE-FALL TIME

(FOR HOMOGENOUS DENSITY INDEP. OF r_0)

DISSIPATIVE COLLAPSE

- TIME NECESSARY FOR NEBULAE (CLOUD) TO COOL SIGNIFICANTLY : t_{cool}

IF $t_{cool} \ll t_{ff}$ RAPID COLAPSE (NO PRESSURE SUPPORT)

THERMAL KINETIC ENERGY IN GAS

$$-2\langle K \rangle = \langle U \rangle$$

GAS MEAN MOLECULAR WEIGHT : μ

GAS N PARTICLES

$M = N \mu m_H$ Mass of NEBULA

$$-2N \frac{1}{2} \mu m_H \langle v^2 \rangle = -\frac{3}{5} \frac{G M^2}{R}$$

μm_H: AVERAGE MASS OF PARTICLE

R: RADIUS OF NEBULA

VELOCITY DISPERSION: $\sigma = \langle v^2 \rangle^{1/2}$

$$\sigma = \left(\frac{3}{5} \frac{GM}{R} \right)^{1/2}$$

VIRIAL TEMPERATURE

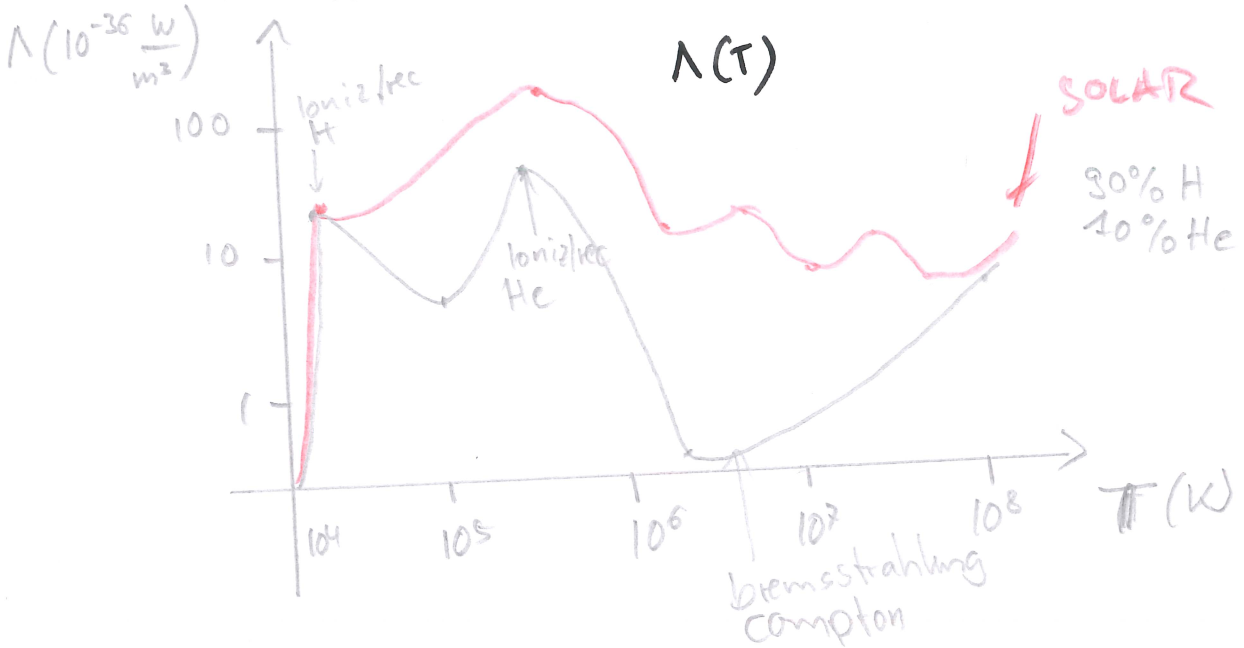
$$\frac{1}{2} \mu m_H \sigma^2 = \frac{3}{2} k T_{virial}$$

$$\Rightarrow \quad T_{virial} = \frac{\mu m_H \sigma^2}{3k}$$

COOLING RATE: (TO DETERMINE RATE AT WHICH ENERGY IS TAKEN AWAY)

$$r_{cool} = n^2 \Lambda(T)$$

- r_{cool} ↑ Energy per time and volume
- n ↑ number density; n^2: interaction pair of particles
- $\Lambda(T)$ ↖ QUANTUM MECHANICAL COOLING FUNCTION



TOTAL ENERGY RADIATED AWAY:

$$r_{cool} V t_{cool} = \frac{3}{2} N k T_{virial}$$

$$\Rightarrow \quad t_{cool} = \frac{3}{2} \frac{k T_{virial}}{n \Lambda}$$

FOR EXAMPLE PROTO-GALACTIC NEBULA:

$$t_{cool} = 8 \, Myr$$

$$\Rightarrow t_{cool} \ll t_{ff} \Rightarrow \text{free fall collapse}$$

- CASE $t_{cool} > t_{ff}$

  ↳ CLOUD TEMPERATURE RAISES

  $\Rightarrow$ INCREASED PRESSURE

  $\Rightarrow$ COLLAPSE HALTS

PROTO GALACTIC CLOUDS: $T \sim 10^6 K$

$n \sim 5 \times 10^4 \, m^{-3}$

UPPER MASS FOR COOLING: $10^{12} M_\odot$

DECREASE TEMPERATURE (HYDROGEN REC.) $T \sim 10^4 K$

$\Downarrow$

$10^8 M_\odot$

$\Rightarrow$ GALAXIES IN MASS RANGE $10^8 - 10^{12} M_\odot$

SMALLEST DWARF ELLIPTICALS (10^8) — MOST MASSIVE SPIRALS (10^{12})

gE'S AND cD EXCEED $10^{12} M_\odot$ (MERGERS)

FORMATION OF THE THICK DISK

- COLLISSION OF GAS CLOUDS
  - ↳ COLLAPSE DISSIPATIVE
- INITIAL ANGULAR MOMENTUM
  (TORQUES FROM NEIGHBOURING CLOUDS)

⇒ COLLAPSING MATERIAL ROTATIONALLY SUPPORTED

↳ SETTLING IN DISK ABOUT GALACTIC CENTRE

FORMATION AT TEMP. $T \sim 10^6 K$

FIELD OF STARS

$$\rho = \rho_0 e^{-z/h}$$

(better $= \rho_0 e^{-|z|/h}$)

MASS DENSITY OF DISK: $\rho(z) = \rho_0 e^{-z/h}$

Gauss's Law: $\oint \vec{g}\, d\vec{A} = -4\pi G M_{in}$

↑ Flux of gravit. field

IF $h \ll d$ (of disk)

↑ diameter

$(\vec{g} \parallel_{anti} \vec{A})$

$$2A \cdot g = 4\pi G M_{in}$$

$$M_{in} = 2\int_0^h \rho_0 e^{-z/h} A\, dz = 1.26 \rho_0 A h$$

$$\Rightarrow g(h) = 2.53\pi \rho_0 G \cdot h$$

Grav. POTENTIAL ENERGY (on particle of MASS m)

$$u(h) = \int_0^h m g(z)\, dz = 1.26\pi G \rho_0 h^2$$

SAME AS $\frac{3}{2} kT = K$

$$\Rightarrow h(T) = \left(\frac{3kT}{2.53\pi G m \rho_0}\right)^{1/2}$$

$$\Rightarrow \rho_{SOLAR} \sim 0.15 \frac{M_\odot}{pc^3} \sim 10^{-20} \frac{g}{cm^3}$$

$$\Rightarrow h(10^6 K) \approx 2.2\,kpc$$

$(m = m_H)$ ← SCALE HEIGHT OF GALAXY $\sim 1kpc$

GALAXY FORMATION IN THE EARLY UNIVERSE

- OVER ABUNDANCE OF BLUE GALAXIES IN TWO DISTANT CLUSTERS
    - BUTCHER – OEMLER EFFECT
        - GALAXIES IN EARLY UNIVERSE BLUER
            - INCREASED STAR FORMATION

- AT EARLY TIMES ELLIPTICAL AND LENTICULAR LESS ABUNDANT THEN SPIRALS

  ⇒ EVOLUTION FROM LATE TO EARLY TYPE

- HUBBLE ULTRA DEEP FIELD (2004) (SLIDE 3')
    - × DISTANT GALAXIES 400 - 800 M yrs after BB
    - × PROTO GALACTIC FRAGMENTS

SLIDE 40,41