Lecture X

THE COSMIC MICROWAVE BACKGROUND (CMB)

DECOUPLING OF PHOTONS AND BARYONS: $z \simeq 1000$

LAST SCATTERING

FLUCTUATIONS IN THE CMB TEMPERATURE

PRIMARY ANISOTROPIES

1) GRAVITATIONAL PERTURBATIONS

PHOTONS FROM HIGH-DENSITY REGIONS AT LAST SCATTERING CLIMB OUT OF POTENTIAL WELLS → SMALLER TEMPERATURE

SACHS-WOLFE EFFECT

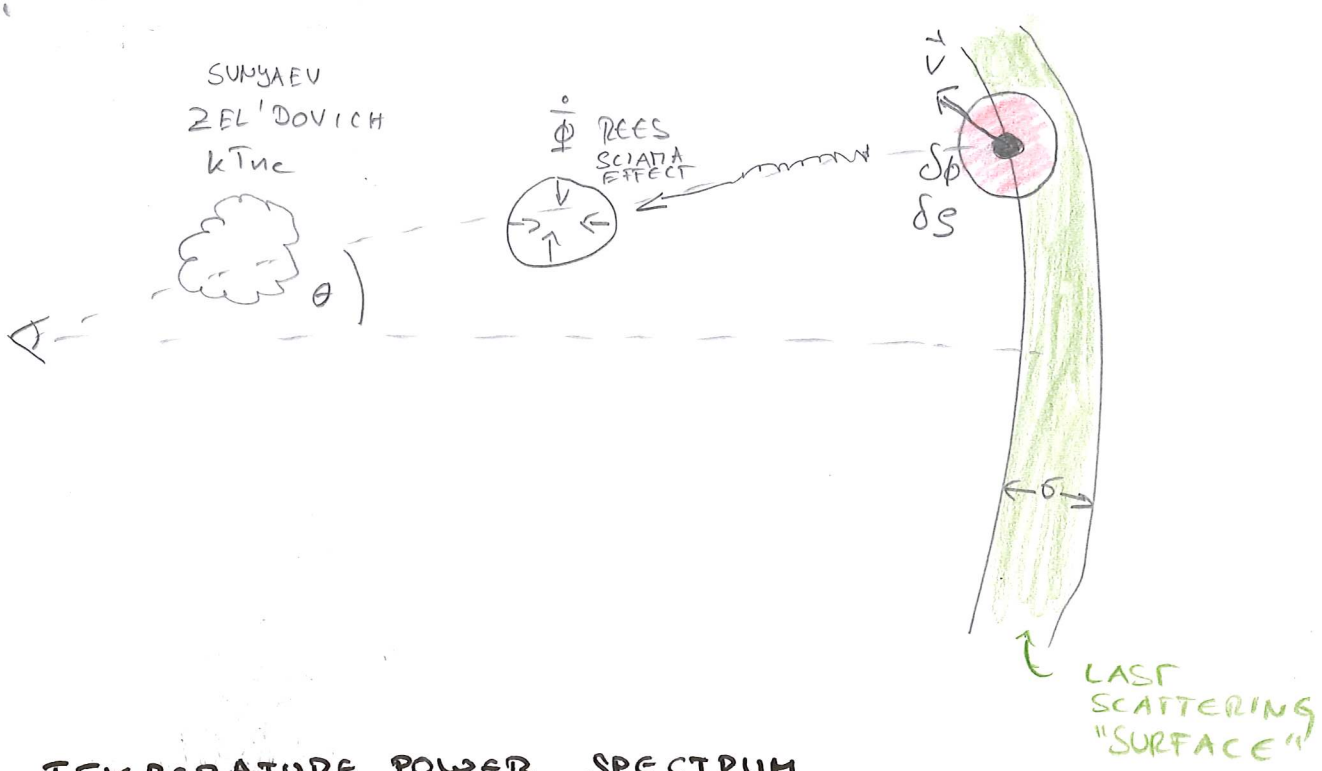
2) INTRINSIC (ADIABATIC) PERTURBATIONS

COUPLING OF MATTER AND RADIATION

↳ COMPRESSION OF RADIATION ⇒ HIGHER TEMPERATURE

3) VELOCITY (DOPPLER) PERTURBATIONS

NON-ZERO VELOCITY OF PLASMA INCREASES TEMPERATURE



TEMPERATURE POWER SPECTRUM

2D FIELD OF RANDOM FLUCTUATIONS IN BRIGHTNESS TEMPERATURE

FOURIER EXPANSION OF TEMPERATURE FLUCTUATIONS (patch of sky of side length L)

$$\frac{\delta T}{T}(\vec{x}) = \frac{L^2}{(2\pi)^2} \int T_k \exp\{-i\,\vec{k}\cdot\vec{x}\}\, d^2k$$

↑ Fourier coeff.

assume small patch of sky → flat

$\vec{x}$: 2-dim pos vector
$\vec{k}$: 2-dim wave vector

DIMENSIONLESS POWER SPECTRUM:

$$\mathcal{J}^2 \equiv \frac{L^2}{(2\pi)^2} 2\pi k^2 |T_k|^2$$

TWO-POINT CORRELATION FUNCTION

$$C(\theta) = \left\langle \frac{\delta T}{T}(\psi) \frac{\delta T}{T}(\psi+\theta) \right\rangle$$

WITH: (Bessel)

$$C(\theta) = \int T^2(\tilde{k})\, J_0(k\theta)\, \frac{dk}{k}$$

(FROM: $\int \exp(ikx\cos\phi)\, d\phi = 2\pi J_0(x)$)

FOR PREDICTING OBSERVED ANISOTROPY FIELD

INTEGRATE OVER LAST SCATTERING SURFACE

ASSUME FLAT SKY (AND FLAT UNIVERSE)

$$\frac{\delta T}{T} = \frac{V}{(2\pi)^3} \iint T_k^{3D}\, e^{-i\vec{k}\vec{r}}\, d^3k\, \underbrace{e^{-\tau}\, d\tau}$$

due to finite thickness of LSS

τ : optical depth

$e^{-\tau}\, d\tau$: prob. of scattering out of line of sight

approximate $e^{-\tau}\, d\tau$ by Gaussian

$$e^{-\tau}\, d\tau \propto \exp\left[-\frac{(r - r_{LS})^2}{2\sigma_r^2}\right] dr$$

r: comoving radius

Since $z_{LS} \approx 1000 \Rightarrow r_{LS} \simeq r_H$

THICKNESS ? σ_r ?

RECOMBINATION

- BELOW $T \approx 1000°K$: IONIZED PLASMA FORMS NEUTRAL ATOMS

SAHA EQUATION

- SINGLE HYDROGEN ATOM

EQUILIBRIUM:

$$\frac{n_e^{free}}{n_e^{bound}} = \frac{P_{free}}{P_{bound}} = \frac{\sum_{free} g_i e^{-E_i/kT}}{\sum_{bound} g_i e^{-E_i/kT}} = \frac{Z_{free}}{Z_{bound}}$$

(Partition)

$g = 2$: electron (spin)

$$Z_{bound} \approx g \exp\left(\frac{\chi}{kT}\right)$$

χ: ionization potential of H

(valid for $kT \lesssim \chi$)

$$Z_{free} = g \frac{V}{(2\pi)^3} \int e^{-\frac{p^2}{2mkT}} \frac{d^3p}{h^3} = \frac{gV}{(2\pi\hbar)^3} (2\pi m kT)^{3/2}$$

volume for each electron: $\sim \frac{1}{n_e}$

(n_e: electron number density)

DEFINE: $x \equiv \frac{n_e}{n}$

$n = n_H + n_p$: nucleon number density

$$n_e^{bound} = n - n_e^{free} = n(1-x)$$

$$\Rightarrow \frac{x}{1-x} = \frac{(2\pi m_e kT)^{3/2} \cdot \cancel{g}}{(2\pi\hbar)^3 \underbrace{n_e^{free}}_{x \cdot n} \cdot \cancel{g}} e^{-\frac{\chi}{kT}}$$

$$\Rightarrow \boxed{\frac{x^2}{1-x} = \frac{(2\pi m_e kT)^{3/2}}{n(2\pi\hbar)^3} e^{-\frac{\chi}{kT}}}$$

$\sim a^{-3}$

$T \sim a^{-1}$

x diverges from 1 when $kT \sim \chi$

NON-EQUILIBRIUM

- single recombination to ground state

$\rightsquigarrow$ production of PHOTON WITH: $\hbar\omega > \chi$

$\updownarrow$

ionizes neutral atom

PHOTONS AT LEAST AS ENERGETIC

AS: $2P \rightarrow 1S$ (Ly α ; $\lambda = 1216$ Å)

(photons can NOT escape UNIVERSE)

HOWEVER: TWO-PHOTON EMISSION

$2S \rightarrow 1S$: FORBIDDEN AT FIRST ORDER

BUT PAIR OF PHOTONS ALLOWS CONSERV OF ENERGY AND ANG. MOM

(ALLOWS $\chi > \lambda_{Ly\alpha}$)

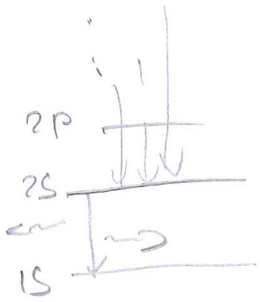
$\hookrightarrow$ RATE VERY DIFFERENT FROM SAHA

TWO-LEVEL ANALYSIS (1S AND 2S)

other chains of recombination can be ignored since it leads to ionizing photons

HAVE TO REACH 2S STATE FIRST:

$$\frac{d(nx)}{dt} = -R(nx)^2 \frac{\Lambda_{2\gamma}}{\Lambda_{2\gamma} + \Lambda_\alpha(T)}$$



n: protons

x: ionization

R: recombination coefficient $\left(\simeq 3\cdot 10^{-19} T^{-1/2} \frac{m^3}{s}\right)$

$\Lambda_{2\gamma}$: two-photon decay rate

$\Lambda_\alpha(T)$: stimulated transition into 2S STATE

R, $\Lambda_\alpha(T)$: functions of temperature (redshift)

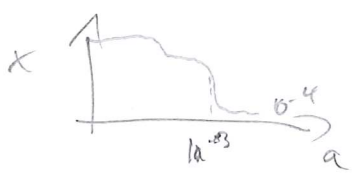
use: $\frac{dt}{dz} \simeq -3.09 \cdot 10^{17} h^{-1} z^{-5/2}\ s$ (in $\frac{\text{FLAT}}{\text{MD}}$)

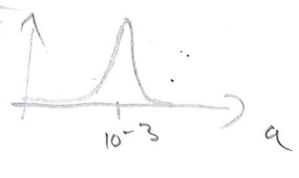
$$\Rightarrow \quad x(z) \propto \frac{h}{\Omega_b h^2}$$

AT LATE TIMES: $\frac{d\ln x}{d\ln z} \simeq 60 \times z\, \Omega_b h$

(NEGLECTS EXPANSION, INVALID FOR $\frac{d\ln x}{d\ln z} < 1$)

(at z a few hundred)





AT $z \approx 1000$

$$x(z) = 2.4 \cdot 10^{-3} \frac{1}{\Omega_b h} \left(\frac{z}{1000}\right)^{12.75}$$

$$\tau = \int n_e x \sigma_T \, dr_{prop}$$

$$\Rightarrow \tau \approx 0.37 \left(\frac{z}{1000}\right)^{14.25}$$

$\Rightarrow e^{-\tau} \frac{d\tau}{dz}$: sharply peaked;
fitted by Gaussian at $\bar{z} \approx 1065$
with width $\sigma_z \approx 80$
(p. 66 lecture) NOTES

photons scattered from range of redshifts

BACK TO CMB

$\sigma_r \approx 7 h^{-1}$ Mpc (FLAT UNIV.)

ASSUME OBSERVED CMB FIELD IN xy-plane

$$\Rightarrow T_k^{3D} \rightarrow T_k^{3D} e^{-\frac{k_z^2 \sigma_r^2}{2}}$$

$$C_{3D} = \int \frac{T_{3D}^2}{4\pi k^3} e^{-i\vec{k}\vec{x}} \, dk_x \, dk_y \, dk_z$$

$$C_{2D} = \int \frac{T_{2D}^2}{2\pi k^2} e^{-i\vec{k}\vec{x}} \, dk_x \, dk_y$$

(NOTE WAVENUMBERS IN 2D AND 3D)

$$\mathcal{J}_{3D}^2 = \frac{V}{(2\pi)^3} 4\pi k^3 \left|T_k^{3D}\right|^2$$

$$\Rightarrow \mathcal{J}_{2D}^2(k) = k^2 \int_0^\infty \mathcal{J}_{3D}^2 \cdot \left(\sqrt{k^2+w^2}\right) e^{-w^2\sigma_v^2} \frac{dw}{(w^2+k^2)^{3/2}}$$

PROJECTION OF 3D SPECTRUM ON 2D

DOMINATED BY $w=0$

2D is smeared version of 3D

SACHS - WOLFE EFFECT

(i) COOLING OF CMB - PHOTONS

$$\frac{\delta T}{T} = \frac{\delta \Phi}{c^2}$$

(ii) TIME DILATION AT LAST SCATTERING

$$\frac{\delta t}{t} = \frac{\delta \phi}{c^2}$$

in MD: $a \propto t^{2/3}$ AND $T \sim a^{-1}$

$$\Rightarrow \frac{\delta T}{T} = -\frac{2}{3}\frac{\delta \phi}{c^2}$$

$$\Rightarrow \text{NET EFFECT:} \quad \frac{\delta T}{T} = \frac{\delta \Phi}{3c^2}$$

RELATION TO DENSITY PERTURBATIONS

$$\nabla^2 \delta\Phi_k = 4\pi G \bar{\rho} \delta_k$$

$$\Leftrightarrow -\frac{k^2}{a^2} \delta\Phi_k = 4\pi G \bar{\rho} \delta_k$$

(MD)

$$T_k = -\frac{(1+z_{LS})}{2} \left(\frac{H_0}{c}\right)^2 \frac{\delta_k(z_{LS})}{k^2}$$

DOPPLER SOURCE TERM

SCATTERING OF PHOTONS BY MOVING PLASMA:

$$\frac{\delta T}{T} = \frac{\delta \vec{v} \cdot \hat{r}}{c}$$

LINEAR PECULIAR VEL. PERTURBATION

$$T_k = -i\sqrt{1+z_{LS}} \left(\frac{H_0}{c}\right) \frac{\delta_k(z_{LS})}{k} \hat{k}\cdot\hat{r}$$

ADIABATIC SOURCE TERM

$$T_k = \frac{\delta_k(z_{LS})}{3}$$

$$\frac{\delta n_\gamma}{n_\gamma} = \frac{\delta s}{s} \quad \text{AND} \quad n_\gamma \propto T^3$$

$$\Rightarrow \quad T_{3D}^2 = \left[(f_A + f_{sw})^2 (k) + f_v^2 (k) \mu^2 \right] \Delta_m^2 (z_{LS})$$

$$(\mu = \hat{k} \cdot \hat{r})$$

WITH: $\quad f_{sw} = - \dfrac{2}{(k D_{LS})^2}$

$$f_v = \frac{2}{k D_{LS}}$$

$$f_A = \frac{1}{3}$$

WITH $\quad D_{LS} = \dfrac{2c}{\Omega_m^{1/2} H_0} (1 + z_{LS})^{-1/2}$

$$\approx 184 \, h^{-1} \, \text{Mpc}$$

SW DOMINATES AT $r \geq 1 h^{-1}$ Gpc
Doppler takes over, but the adiabatic effects dominate

SMALL SCALES $\longrightarrow$ NEXT WEEK